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Type II NS Five-Branes: Non Critical Strings & their Topological Sectors^{*}

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Abstract

The near-horizon geometry of parallel NS5-branes is described by an exact superconformal 2d field theory based on $\mathcal{K}^{10} \equiv W_k^4 \times \mathcal{M}^6$, with \mathcal{M}^6 a flat 5+1 space time and $W_k^4 \equiv U(1) \otimes SU(2)_k$, a four dimensional background with non-trivial dilaton. The ten-dimensional “BULK” spectrum of excitations of \mathcal{K}^{10} can be derived combining unitary representations of the $\mathcal{N} = 4$ superconformal theory of W_k^4 and \mathcal{M}^6 in a modular-invariant way. All Bulk states are massive and belong to the long representations of the $N_6 = 2$ space-time supersymmetry. The NS5-brane states, propagating on \mathcal{M}^6 , belong to the short representations of $N_6 = 2$. Both the bulk and the brane spectrum is derived using the powerful worldsheet technics of the $\mathcal{N} = 4$ superconformal theories. We claim that both bulk and brane states are necessary to well define the theory. The non abelian $U(n)$ or $SO(2n)$ structure of the 5-brane fields follows from the fusion coefficients appearing in the correlation functions involving $SU(2)_k$ conformal fields.

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1 Introduction

The study of string propagation in non-trivial gravitational backgrounds can provide a better understanding of quantum gravitational phenomena. Non-trivial classical string backgrounds can be obtained by two different methods. The first makes use of a two-dimensional σ -model where the space-time backgrounds correspond to field-dependent coupling constants. The vanishing of the corresponding β -functions is identified with the background field equations of motion in the target space [1]. The second approach consists of replacing the free space-time coordinates by non-trivial (super) conformal systems, which, in the semiclassical limit, can be interpreted as describing a string propagation in non-trivial space-time. The two methods are useful and complementary. The σ -model approach provides a clear geometric interpretation, but it has the disadvantage of the α' -expansion, which is valid only when all curvetures and derivatives on space-time fields are small. The conformal field theory approach takes into account all orders in α' automatically and has the main advantage of deriving exact string vacua. A special class of exact solutions are based on $N = 4$ superconformal systems [2, 3, 4, 5] where the degrees of freedom of the ten supercoordinates form three superconformal systems

$$\{\hat{c}\} = 10 = \{\hat{c} = 2\} + \{\hat{c} = 4\}_1 + \{\hat{c} = 4\}_2 . \quad (1.1)$$

The $\hat{c} = 2$ two free superfields. The time-like supercoordinate and a space-like supercoordinates. The remaining eight supercoordinates appear in groups of four in $\{\hat{c} = 4\}_1, \{\hat{c} = 4\}_2$. Both $\{\hat{c} = 4\}_A$ systems exhibit $N = 4$ superconformal symmetry of the Ademollo et al [2, 3, 4, 5]. The non-triviality of these solutions follows from the fact that there exist realizations of the $\hat{c} = 4, N = 4$ superconformal systems that are based on geometrical and topological non-trivial spaces [3, 4, 5], other than the T^4/Z_2 orbifold and the K_3 compact manifold.

The first subclass is characterized by two integer parameters k_1, k_2 , which are the levels of two $SU(2)$ group manifolds. For weakly curved backgrounds (large k_A) these solutions can be interpreted in terms of a ten-dimensional, but topologically non-trivial, target space of the form $R^4 \otimes S^3 \otimes S^3$. In the special limit $k_2 \rightarrow \infty$ one obtains the semi-wormhole solution of Callan, Harvey and Strominger [6], based on a six-dimensional flat background, combined with a four-dimensional space $W_k^{(4)} \equiv U(1) \otimes SU(2)_k$.

The underlying superconformal field theory includes a supersymmetric $SU(2)_k$ WZW model describing the three coordinates of S^3 as well as a non-compact dimension with a background charge, describing the scale factor of the sphere. $M^6 \otimes W_k^{(4)}$ corresponds to the gravitational *back reaction* of $n = k + 2$ NS five-branes [6, 7, 8].

A second subclass of solutions is based on a different realization of the $N = 4$ superconformal system with $\hat{c} = 4$, it replaces the $W_k^{(4)}$ space by:

$$\Delta_k^{(4)} \equiv \left\{ \left[\frac{SU(2)}{U(1)} \right]_k \otimes \left[\frac{SL(2, R)}{U(1)} \right]_{k+4} \right\}_{\text{SUSY}} . \quad (1.2)$$

A gauged supersymmetric WZW model with $\hat{c}[\Delta_k^{(4)}] = 4$ for any value of k [4, 5]. The choice of the levels k and $k + 4$ is necessary for the existence of an $N = 4$ symmetry with $\hat{c} = 4$. Another subclass of solutions is obtained using the *T-dual* [9, 10] of $W_k^{(4)}$, $C_k^{(4)}$ [4, 5]

$$C_k^{(4)} \equiv \left(\frac{SU(2)}{U(1)} \right)_k \otimes U(1)_R \otimes U(1)_Q \quad (1.3)$$

with a background charge $Q = \sqrt{2/(k+2)}$ in one of the two coordinate currents ($U(1)_Q$). The other free coordinate ($U(1)_R$) is compactified on a torus with radius $R = \sqrt{2k}$.

2 10-d Target Spaces, via $N = 4$, $\hat{c} = 4$ Conformal Block Construction [4, 5]

Having at our disposal non-trivial $N = 4$, $\hat{c} = 4$ superconformal systems, we can use them as building blocks to obtain new classes of *exact* and *stable* string solutions around non-trivial backgrounds in both type-II and heterotic superstrings.

$$\mathcal{M}^2 \otimes W_{k_1} \otimes W_{k_2} \quad \textbf{9-branes or 2x5-branes}$$

$$\mathcal{M}^6 \otimes W_k \quad \textbf{5-branes,}$$

$$\mathcal{M}^2 \otimes C_{k_1} \otimes C_{k_2} \quad \textbf{T-duals of 9- or 2x5-branes}$$

$$\mathcal{M}^6 \otimes C_k \quad \textbf{T-dual of 5-branes,}$$

$$\mathcal{M}^2 \otimes C_{k_1} \otimes W_{k_2} \quad \textbf{T-duals of 9- or 2x5-branes}$$

$$\mathcal{M}^2 \otimes C_{k_1} \otimes \Delta_{k_2} \quad \textbf{T-duals of 9- or 2x5-branes}$$

$$\mathcal{M}^2 \otimes \Delta_{k_1} \otimes W_{k_2} \quad \textbf{T-duals of 9- or 2x5-branes}$$

$$\mathcal{M}^2 \otimes \Delta_{k_1} \otimes \Delta_{k_2} \quad \textbf{T-duals of 9- or 2x5-branes}$$

$$\mathcal{M}^6 \otimes \Delta_k \quad \textbf{T-dual of 5-branes,}$$

All the above superconformal systems correspond either to *Non-critical Superstrings*[11, 4, 5] and/or to a *Bulk 10d Superstring theory with branes* [6, 7, 4, 5, 8] or to a *background solution of the $SU(2) \times SU(2)$ Gauged Supergravities*[12, 5].

3 The Spectrum of String Excitations in the Bulk

The explicit realizations of the $N = 4$ algebra in terms of known conformal field theories, allows us to compute the spectrum of string excitations in any of the background solutions given in the previous Section.

The $N = 4$ superconformal symmetry implies the existence of space-time supersymmetry [13] in the corresponding non-trivial target spaces, and thus guarantees the stability of the theory.

In all constructions, the total number of space-time supersymmetries is reduced by a factor of 2 with respect to the flat (toroidal) compactifications. In the context of the σ -model approach, the $N = 4$ spectral flows are related to the number of covariantly constant spinors admitted by the corresponding non-trivial target spaces [6].

The reduction of space-time supersymmetries by a factor of 2 in W^4 non-trivial space is due to the torsion and background charge term appearing, in the supercurrents [5]. $Q^2 = 2/(k + 2)$

$$T = -\frac{1}{2} \left[Q^2 J_i^2 + J_4^2 - \Psi_a \partial \Psi_a + Q \partial J_4 \right] ,$$

$$G_4 = Q \left(J_i \Psi_i + \frac{1}{3} \epsilon_{ijl} \Psi_i \Psi_j \Psi_l + \partial \Psi_4 \right) + J_4 \Psi_4 ,$$

$$\begin{aligned}
G_i &= Q (J_i \Psi_4 - \epsilon_{ijl} J_j \Psi_l + \epsilon_{ijl} \Psi_4 \Psi_j \Psi_l - \partial \Psi_i) - J_4 \Psi_i , \\
S_i &= \frac{1}{2} \left(\Psi_4 \Psi_i + \frac{1}{2} \epsilon_{ijl} \Psi_j \Psi_l \right) .
\end{aligned} \tag{3.4}$$

The global existence of the (chiral) $N = 4$ superconformal algebra implies a universal GSO projection that generalizes the one of the free-field realization and it is responsible for the existence of space-time supersymmetry. The reduction of space-time supersymmetries by a factor of 2 implies an extra GSO projection involving the $SU(2)_k$ spin j and the spins of the $SO(4)_{\Psi_I} \equiv SU(2)_+ \times SU(2)_-$ constructed by the 2d fermions $\Psi_I, I = 1, 2, 3, 4$.

3.1 The spectrum in a five-branes background

The basic rules to construct the bulk spectrum are similar to that of, the orbifold construction [14], the free 2-d fermionic constructions [15], and the Gepner construction [16]. One combines in a modular invariant way the world-sheet degrees of freedom consistently with unitarity and spin-statistics of the string spectrum. The six non-compact coordinates, together with the reparametrization ghosts (\mathbf{b}, \mathbf{c}) , provide a contribution to the (type-II) partition function:

$$Z_B[F^{(6)}; (\mathbf{b}, \mathbf{c})] = \frac{\text{Im} \tau^{-2}}{\eta^4(\tau) \bar{\eta}^4(\bar{\tau})} . \tag{3.5}$$

The contribution of the \mathcal{M}^6 world-sheet fermions together with the β and γ super-reparametrization ghosts is:

$$Z_F[M^6; (\beta, \gamma)] = (-)^{\alpha+\beta} \frac{\theta^2(\alpha)}{\eta^2(\tau)} (-)^{\bar{\alpha}+\bar{\beta}} \frac{\bar{\theta}^2(\bar{\alpha})}{\bar{\eta}^2(\bar{\tau})} . \tag{3.6}$$

The Neveu-Schwarz (NS, \overline{NS}) sectors correspond to $\alpha, \bar{\alpha} = 0$ and the Ramond (R, \bar{R}) sectors correspond to $\alpha, \bar{\alpha} = 1$.

Then, one must combine the above M^6 characters with those of $W_k^{(4)}$ [5]:

(i) the $SU(2)_k$ characters, $(\chi_k^L, L = 1, 2, \dots, k)$,

(ii) the $U(1)_Q$ Liouville characters,

(iii) the $SU(2)_+$, $(\chi_+^l, l = 0, 1)$,

(iv) the $SU(2)_-$, $(\chi_-^l, l = 0, 1)$.

- The $U(1)_Q$ Liouville characters:

i) *Continuous Representations* generated by the lowest-weight operators:

$$e^{\beta X_L} ; \quad \beta = -\frac{1}{2}Q + ip , \quad (3.7)$$

with *positive conformal weights*

$$h_p = \frac{Q^2}{8} + \frac{p^2}{2}.$$

The fixed imaginary part in the momentum $iQ/2$ of the plane waves, is due to the non-trivial dilaton motion.

ii) *Discrete Representations*. They correspond to lowest-weight operators with $\beta = Q\tilde{\beta}$ real, leading to *negative conformal weights*

$$h = -\frac{1}{2}\tilde{\beta}(\tilde{\beta} + 1)Q^2 = -\frac{\tilde{\beta}(\tilde{\beta} + 1)}{k + 2} \quad (3.8)$$

Both categories of Liouville representations give rise to *unitary representations* of the $N = 4$, $\hat{c} = 4$ system $W_k^{(4)}$, once they are combined with the remaining degrees of freedom.

The *continuous representations* form *long* (massive) representations of $N = 4$ with conformal weights larger than the $SU(2)$ spin, $h > S$ [2, 5]. All bulk states belong to long representations of the $N = 4$.

On the other hand the discrete representations form *short* representations of $N = 4$ with $h = S$, [2, 5] and thus β take only a *finite number of values* [5]

$$-(k + 2)/2 \leq \tilde{\beta} \leq k/2 . \quad (3.9)$$

In fact, their locality with respect to the $N = 4$ operators implies:

$$S = \frac{1}{2}, \quad \tilde{S} = \frac{1}{2} : \quad \tilde{\beta} = -(j + 1) ,$$

$$S = 0, \quad \tilde{S} = 0 : \quad \tilde{\beta} = j, \quad (3.10)$$

The vertex operators corresponding to the discrete representations take the form

$$R[S, j, \tilde{\beta}] = V_{\Psi}(S) \bar{V}_{\tilde{\Psi}}(\tilde{S}) \chi_k(j) \bar{\chi}_k(j) e^{\beta X_L}, \quad (3.11)$$

have fixed conformal weights in the -1 picture, $(h_L, h_R) = (\frac{1}{2}, \frac{1}{2})$, giving rise to massless states belonging to the short representations of the $N = 4$ [5]. They correspond to the 5-Brane States propagating on \mathcal{M}^6 [8]. In the massless spectrum there are either $U(k+2)$ or $SO(k+4)$ [8]. In the first case 2j integer ($SU(2)_k$ 2d WZW-model) while in the second case, j takes integer values ($SO(3)_{k/2}$ 2d WZW-model).

All Bulk States are massive with minimal mass $m_{\min}^2 = Q^2/4$ due to the linear dilaton [17] and torsion [5, 18]. A modular invariant partition function for the BULK states (k even) is [5]:

$$Z_W = \frac{\text{Im } \tau^{-5/2}}{\eta^5 \bar{\eta}^5} \frac{1}{8} \sum_{\alpha, \beta, \bar{\alpha}, \bar{\beta}, \gamma, \delta} (-)^{\alpha+\beta} \frac{\theta^2(\frac{\alpha}{\beta})}{\eta^2} \frac{\theta^2(\frac{\alpha+\gamma}{\beta+\delta})}{\eta^2} \times \\ (-)^{\bar{\alpha}+\bar{\beta}} \frac{\bar{\theta}^2(\frac{\bar{\alpha}}{\bar{\beta}})}{\bar{\eta}^2} \frac{\bar{\theta}^2(\frac{\bar{\alpha}+\gamma}{\bar{\beta}+\delta})}{\bar{\eta}^2} (-)^{\delta(\alpha+\bar{\alpha}+\frac{k}{2}\gamma)} Z_k[\frac{\gamma}{\delta}] \quad (3.12)$$

The first factor is the contribution of the non-compact coordinates and that of the Liouville mode.

$Z_k[\frac{\alpha}{\beta}]$ defines appropriate character combinations of $SU(2)_k$. Under modular transformations [5]:

$$Z_k[\frac{\alpha}{\beta}] = \sum_{L=0}^k e^{i\pi\beta L} \chi_k^L \bar{\chi}_k^{L+\alpha(k-2L)},$$

α, β can be either 0 or 1.

$$\tau \rightarrow \tau + 1 \quad : \quad Z_k[\frac{\alpha}{\beta}] \longrightarrow e^{-i\pi\frac{k}{2}\alpha^2} Z_k[\frac{\alpha}{\beta+\alpha}]$$

$$\tau \rightarrow -1/\tau \quad : \quad Z_k[\frac{\alpha}{\beta}] \longrightarrow e^{i\pi k\alpha\beta} Z_k[\frac{\beta}{\alpha}].$$

the β and $\bar{\beta}$ summations give rise to universal (left- and right-moving) GSO projections, which imply the existence of space-time supersymmetry. The summation over δ gives rise to an additional projection, which correlates the $SU(2)_-$ (left and right) spin

together with the spin of $SU(2)_k$. It reduces the number of supersymmetries by a factor of 2.

$$2\tilde{S}_2 + 2\bar{\tilde{S}}_2 + L + \frac{k}{2}\gamma = \text{even}. \quad (3.13)$$

In the $\gamma = 0$ sector, the lower-lying states have (left and right) mass-squared $Q^2/8$ and $L = 0$. It is convenient to classify the states in the context of a six-dimensional theory. The lower-lying states come from the gravitational supermultiplet of the six-dimensional $N = 2$ supergravity:

$$(|\Psi^\mu > + |(\text{spin}\Psi^\mu)_-(\text{spin}\Psi^I)_+ >) \otimes (|\bar{\Psi}^\mu > + |(\text{spin}\bar{\Psi}^\mu)_-(\text{spin}\bar{\Psi}^I)_+ >) e^{ip_\mu X^\mu + (ip - \frac{Q}{2})X_L}$$

together with four vector multiplets:

$$(|\Psi^I > + |(\text{spin}\Psi^\mu)_+(\text{spin}\Psi^I)_- >) \otimes (|\bar{\Psi}^I > + |(\text{spin}\bar{\Psi}^I)_+(\text{spin}\bar{\Psi}^I)_- >) e^{ip_\mu X^\mu + (ip - \frac{Q}{2})X_L}.$$

As expected from the effective field theory point of view, their mass-squared $Q^2/8$ is due to the dilaton motion for bosons, and to the non-trivial torsion for fermions.

The ($\gamma = 1$) (Twisted) sector contains states with (left and right) mass-squared always larger than $(k-2)/16$. For any $k > 2$, the twisted states have masses larger than $Q^2/8$ and the lower mass spectrum comes always from the $L = 0$ states contained in the untwisted sector. In that sense $k = 2$ is an exceptional case [5], since the lower-lying twisted states are massless with $L = \bar{L} = k/2 = 1$. These states form massless unitary representations (short) of the $N = 4$ $\hat{c} = 8$.

All other *short* multiplets are coming from the n-NS5-Branes propagating in the six dimensional target space.

$$R[S, j, \tilde{\beta}] e^{ik_\mu X^\mu}, \quad \text{with} \quad (h_L, h_R) = (\frac{1}{2}, \frac{1}{2}). \quad (3.14)$$

4 Conclusions

Superstring solutions in the semiclassical limit define background solutions of the extended supergravities. This limit turns out to be very useful regarding the study of the string-induced low-energy theories, as well as the study of physics in weakly curved domains of space time. The supergravity field theory picture fails when the involved curvetures are strong. It is then necessary to go beyond the semiclassical limit and

work directly on the string level, using the powerful techniques of the underlying two-dimensional (super) conformal field theory.

For a generic string background the stringy approach is at present non-accessible. It is possible to go further in the stringy direction for some special backgrounds based on the $N = 4$ superconformal symmetry. Some of the constructions are connected to the non-critical strings and to some stable solutions of the gauged supergravities.

The ten-dimensional “bulk” spectrum of excitations can be derived combining unitary representations of the $N = 4$ superconformal theory in a modular-invariant way. In the case of W_k^4 constructions, these representations are expressed in terms of the well-known $SU(2)$ characters, while in all other constructions one uses also the characters of some compact $SU(2)/U(1)$ and/or non-compact $SL(2, R)/U(1)$ parafermions [16, 19]. In the five-brane construction all bulk states are massive as soon as k is large (Massive Representation of $N=4$). When $k = 2$, there extra massless “twisted” states other than the Five-brane States ($U(n)$ or $SO(2n)$ vector multiplet states).

The Non-propagating brane states correspond to Liouville Discrete representations with negative conformal weight. The non abelian $U(n)$ and $SO(2n)$ structure of the 5-brane fields, $\langle R(j_1)R(j_2)R(j_3) \rangle \sim F_{j_1, j_2, j_3}$, follows from the correlation of the $SU(2)_k$ vertex operators [8].

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